Measurements of Higher Order Flow Harmonics in Au + Au Collisions at $\sqrt{s_{NN}} = 200$ GeV

The production of particles in heavy ion collisions at the Relativistic Heavy Ion Collider is anisotropic in directions transverse to the beam. For low momentum particles ($p_T \leq 3 \text{ GeV}/c$), this anisotropy is understood to result from hydrodynamically driven flow of the quark-gluon plasma (QGP) [1–5]. The strength of the flow is measured as Fourier coefficients $v_n = \langle e^{i n (\phi - \Psi_{RP})} \rangle$, $n = 2, 4, \ldots$ where $\phi$ is the azimuthal angle of an emitted particle around the $z$ axis defined by the beam; $\Psi_{RP}$ is the azimuth of the reaction plane defined by the beam direction and the impact vector between the colliding nuclei. The brackets denote averaging over particles and events. The reaction plane is not measurable directly a priori, so the Fourier coefficients are determined with respect to the estimated participant event planes [1]. Earlier measurements have focused on the even-order anisotropies $v_2$ and $v_4$, evaluated with respect to an event plane $\Psi_2$, determined from the $n = 2$ correlation.

The $v_2(v_4)$ values obtained this way for a broad range of $p_T$ and centrality have been used to extract the specific viscosity $\eta/s$ (the ratio of shear viscosity $\eta$ to entropy density $s$) of the hot and dense nuclear matter via hydrodynamic model comparisons [6–10]. These model comparisons, which incorporate the dynamic evolution of an early-stage strongly coupled QGP, together with a late-stage hadronic gas, show an ambiguity for very different values of $4\pi \eta/s \approx 2$ and $4\pi \eta/s \approx 1$, the latter being a conjectured lower bound for the specific viscosity [11]. Specifically the two values correspond to two equally successful parameter sets, each including different estimates of the initial state anisotropy (parameterized as “eccentricity” see below) [7,8,12], which dominate the associated uncertainty in these models. The lower bound value is obtained with a standard Glauber Monte Carlo (Glauber-MC) model [13,14] of the initial state which results in smaller initial elliptical eccentricity and thus needs less viscosity to reproduce the measured final state particle anisotropy. The higher value $4\pi \eta/s \approx 2$, corresponds to a larger initial eccentricity in the color-glass condensate inspired Monte Carlo Kharzeev-Levin-Nardi (MC-KLN) model [15–17] of the initial state.

Recently, significant attention has been given to the study of the influence of initial geometry fluctuations of the initial state anisotropy [18] which are typically quantified by higher-order generalized “eccentricities” $e_n$ [18,19]. The goal has been to understand how such fluctuations induce anisotropic particle emission, characterized by $v_n$ (for odd and even $n$)

$$\frac{dN}{d\phi} \approx 1 + \sum_{n=1}^{\infty} 2v_n \cos(n(\phi - \Psi_n)), \quad (1)$$

where $v_n = \langle \cos(n(\phi - \Psi_n)) \rangle$, $n = 1, 2, 3, \ldots$ and the $\Psi_n$ are the generalized participant event planes at all orders for each event. These recent developments suggest that measurements of $v_n$, especially for $n = 3$, can yield important additional constraints that provide a more precise estimate of $\eta/s$, as well as resolve the correct eccentricity model.

Here we present results for differential measurements following Eq. (1), for Au + Au collisions at $\sqrt{s_{NN}} = 200 \text{ GeV}$. We first show how the measured event planes correlate across large rapidity gaps, and then show resulting $v_n$ moments for midrapidity particles relative to those planes. The results are derived from $\sim 3.0 \times 10^7$ Au + Au events obtained with the PHENIX detector [20] during the 2007 running period. Collision centrality (related to impact parameter) and number of participating nucleons ($N_{\text{part}}$) are estimated as in [9] through comparisons of detected multiplicity in beam-beam counters (BBCs) [21] with a Glauber-MC calculation. Event planes were determined using three separate detector systems: the same BBCs, reaction-plane detectors (RXNs) [22], and muon-piston

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Flow coefficients $v_n$ for $n = 2, 3, 4$, characterizing the anisotropic collective flow in Au + Au collisions at $\sqrt{s_{NN}} = 200 \text{ GeV}$, are measured relative to event planes $\Psi_n$, determined at large rapidity. We report $v_n$ as a function of transverse momentum and collision centrality, and study the correlations among the event planes of different order $n$. The $v_n$ are well described by hydrodynamic models which employ a Glauber Monte Carlo initial state geometry with fluctuations, providing additional constraining power on the interplay between initial conditions and the effects of viscosity as the system evolves. This new constraint can serve to improve the precision of the extracted shear viscosity to entropy density ratio $\eta/s$.

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calorimeters (MPCs). Each detector system has a north (south) component to measure at forward (backward) rapidity. The absolute pseudorapidity ($\eta'$) coverages for these detectors are $3.1 < |\eta'_{\text{NN}}| < 3.9$, $1.0 < |\eta'_{\text{KNN}}| < 2.8$, $3.1 < |\eta'_{\text{MPC}}| < 3.7$. The PHENIX drift and pad chambers [23] were used for charged particle tracking and momentum reconstruction with azimuthal angle coverage $\varphi = \pi$ rad in the central region ($|\eta'| \leq 0.35$).

To estimate the event plane $\Psi_n$ in each detector, we generalize to all orders $n$ our earlier procedure for event-plane determination (see [9] and especially definitions in [24]). For each event-plane detector we evaluate $\tan(n\Phi_n) = \sum w_i \sin(n\phi_i) / \sum w_i \cos(n\phi_i)$ for the $\Psi_n$ sub-event estimator $\Phi_n$, where the $\phi_i$ are the azimuths of elements in that detector and the weights $w_i$ reflect the energy or multiplicity in that element. Acceptance corrections [24] for imperfect detector efficiency were employed to ensure a flat (azimuthally independent) event-plane distribution, as required by symmetry considerations. In general, the hit distributions sample virtually all momenta.

To measure $v_n$, the azimuth $\phi$ of each particle is correlated with the $\Psi_n$ via Eq. (1). The measured $v_n[\Psi_n] = \langle \cos(n\Phi_n - \Phi_n^\text{av}) \rangle / \text{Res}(\Psi_n)$, where $\Phi_n^\text{av}$ is the average of the $\Phi_n$ for north and south subevents and where the denominator $\text{Res}(\Psi_n)$ represents a resolution factor described in [24]. This factor corrects $v_n$ for the event-by-event dispersion of the $\Phi_n$. Its magnitude can be estimated via the two and three subevents method [9] in which the correlation between $\Phi_n$ from different subevents is measured. The strength of this correlation is generally quantified as $\langle \cos(n\Phi_n - \Phi_m^\text{av}) \rangle$ for subevents $A, B$, which measures the cosine of the dispersion of the $\Phi_n$ estimator with respect to the true $\Psi_n$.

Figure 1 shows the centrality dependence of this correlation strength $\langle \cos(n\Phi_n - \Phi_m^\text{av}) \rangle$ for subevent combinations ($A, B$) involving different event-plane detectors with $\Delta \eta' \sim 5$ and $\Delta \eta' \sim 7$. The raw correlations are presented as measured; however, the magnitudes are specific to the PHENIX detectors involved. The systematic uncertainties (not shown) for these correlations are of similar relative size to those for $v_n[\Psi_n]$ discussed below. The uncertainties are correlated across centrality and $n$ such that the relative size of these event-plane correlations can be compared. The magnitudes for the odd parity quantities $\langle \sin(n\Phi_n - \Phi_m^\text{av}) \rangle$, which should vanish, are found to be consistent with zero for all centrality, $j$, and $\Phi$ combinations. Figure 1 panels (a) and (b) show the two subevent correlations for $m = n$; (c) and (d) show the two subevent correlations for $m \neq n$. The negative correlation indicated in (a) for $n = 1$ is due to the well-known antisymmetric pseudorapidity dependence (sign change about midrapidity) of sideways flow $v_1$, as well as momentum conservation [2]. Positive subevent correlations are indicated in (a) and (b) for $\Psi_{2,3,4}$, with sizable magnitudes for $\Psi_{2,3}$ and much smaller values for $\Psi_4$.

The subevent correlations $\langle \cos(n\Phi_n - \Phi_m^\text{av}) \rangle$ for $n \neq m$ are also of interest. Figure 1(c) confirms the expected correlation between $\Psi_2$ and $\Psi_3$, due to sideways flow, as well as that between $\Psi_2$ and $\Psi_4$ [24]. By contrast, Fig. 1(d) shows that there is no significant correlation observed between $\Psi_2$ and $\Psi_3$, a result which is independent of the detectors used. The order $j = 6$ is chosen to account for the $n$ multiplet of directions $(2\pi/n)$ of $\Psi_2$ and $\Psi_3$. The absence of this correlation suggests that the fluctuations for $\Psi_3$ about $\Psi_2$ are substantial. This is well reproduced by Glauber modeling [25,26] and therefore supports an initial state fluctuation origin of $\Psi_3$ and $\nu_3$.

A small correlation between $\Psi_3$ and $\Psi_1$ is indicated in Fig. 1(d). While such a correlation seems to be at odds with the absence of a $\Psi_2 - \Psi_3$ correlation [Fig. 1(d)], we note that $\Psi_1 - \Psi_3$ correlations need not contribute to a residual contribution to $\Psi_2 - \Psi_3$ correlations through $\Psi_1$. That is, $\Psi_1$ could correlate with $\Psi_3$ and $\Psi_2$ in exclusive event classes. Correlations involving the PHENIX zero-degree calorimeter, which measures the $n = 1$ spectator neutron event plane [24] at $|\eta'| > 6.5$ indicate that this correlation has some degree of $\eta'$ asymmetry. We defer further investigation of these correlation subtleties to future work.

Figure 2 shows results for the midrapidity $v_n[\Psi_n]$ for tracks in the central arms as a function of $p_T$ for different centralities. RXN-defined event planes, which have the
FIG. 2 (color online). $v_n(\Psi_n)$ vs $p_T$ measured via the reaction-plane method for different centrality bins; 0%-10% are the most central collisions. Shaded (gray and pink) and hatched (blue) areas around the data points indicate sizes of systematic uncertainties. The curves in panels (b) and (d) are predictions for $v_2(\Psi_2)$ and $v_3(\Psi_3)$ from two hydrodynamic models, both using Glauber initial conditions and $4\pi \eta/s = 1$, Alver et al. [27] and Schenke et al. [32].

best resolution, are employed. The systematic uncertainties for these measurements were estimated by detailed comparisons of the results obtained with the RXN, BBC, and MPC event-plane detectors and subevent selections. They are $\sim 3\%$, $\sim 8\%$ and $\sim 20\%$ for $v_2(\Psi_2)$, $v_3(\Psi_3)$, and $v_4(\Psi_4)$, respectively, for midcentral collisions and increase by a few percent for more central and peripheral collisions. Through further comparison of the results obtained with the RXN, BBC, and MPC event-plane detectors, pseudorapidity dependent nonflow contributions that may influence the magnitude of $v_n(\Psi_n)$, such as jet correlations, were shown [9] to be much less than all other uncertainties for $v_2(\Psi_2)$ and $v_4(\Psi_4)$.

The $v_n(\Psi_n)$ values shown in Fig. 2 increase with $p_T$ for most of the measured range, and decrease for more central collisions. The $v_2(\Psi_2)$ increases as expected from central to semiperipheral collisions, following the expected increase of $e_n$ with impact parameter [19,27,28]. The $v_3(\Psi_3)$ and, albeit with less statistical significance, also the $v_4(\Psi_4)$ appear to be much less centrality dependent, with $v_3$ values comparable to $v_2(\Psi_2)$ in the most central events. This behavior is consistent with Glauber calculations of the average fluctuations of the generalized “triangular” eccentricity $e_3$ [25,26]. The Fig. 2 panels (b) and (d) show comparisons of $v_2(\Psi_2)$ and $v_3(\Psi_3)$ to results from hydrodynamic calculations. The $p_T$ and centrality trends for both $v_2(\Psi_2)$ and $v_3(\Psi_3)$ are in good agreement with the hydrodynamic models shown, especially at $p_T$ below $= 1$ GeV/$c$.

Figure 3 compares the centrality dependence of $v_2(\Psi_2)$ and $v_3(\Psi_3)$ with several additional calculations, demonstrating both the new constraints the data provide and also the robustness of hydrodynamics to the details of different model assumptions for medium evolution. Alver et al. [27] use relativistic viscous hydrodynamics in $2 + 1$ dimensions. Fluctuations are introduced for two different initial conditions. For Glauber initial conditions, the energy density distribution in the transverse plane is proportional to a superposition of struck nucleon and binary-collision densities; in MC-KLN initial conditions the energy density profile is further controlled by the dependence of the gluon saturation momentum on the transverse position [16,17].

The Glauber-MC and MC-KLN initial state models are paired with the values $4\pi \eta/s = 1$ and 2, respectively, to reproduce the measured $v_2(\Psi_2)$ [8]. The viscosity difference compensates for the $\sim 20\%$ difference between the initial $e_2$ values associated with each model. The two models have similar $e_3$, and thus the larger viscosity needed with MC-KLN calculations to match $v_2$, leads to a much lower $v_3$ than obtained with Glauber MC calculations. Consequently, our measurement of $v_3(\Psi_3)$ helps to...
disentangle viscosity and initial conditions. The efficacy of these 2 + 1 hydrodynamic results for Glauber initial conditions are confirmed by further calculations with different model assumptions. Petersen et al. [29] determine a Glauber initial state event by event, translating through preequilibrium with the UrQMD transport model [30,31], then evolving the medium with ideal QGP hydrodynamics \((\eta/s = 0)\), and finally switching to a hadronic cascade (which has an effective viscosity) as regions become dilute. B. Schenke et al. [32] use event-by-event Glauber initial conditions, evolved with relativistic viscous 3 + 1 dimensional hydrodynamics with 4πη/s = 1.

All of these models are compared with \(v_2(\Psi_2)\), and \(v_3(\Psi_3)\) data as a function of \(N_{\text{part}}\) in two \(p_T\) bins. All calculations describe \(v_2(\Psi_2)\) well at \(p_T = 0.75 \text{ GeV}/c\). Deviations from hydrodynamics should be expected in peripheral collisions, where nonequilibrium effects may be large. At higher \(p_T\), differences between the calculations become more apparent. All models still agree with \(v_2(\Psi_2)\), including MC-KLN initial conditions. However, the lower panels of Fig. 3 show the constraining power of \(v_3(\Psi_3)\) and that the calculated results from viscous hydrodynamics, with MC-KLN initial conditions and 4πη/s = 2, lie significantly below the data. This is more apparent in the higher \(p_T\) bin, even in the most central collisions. Therefore, our comparisons suggest that the combination of the current implementation of the MC-KLN initial conditions in concert with 4πη/s = 2 is disfavored by our new \(v_3(\Psi_3)\) measurements. This may suggest that the MC-KLN implementation or its application needs to be reevaluated (see [33]), but it does not necessarily imply that a color-glass condensate initial state is disfavored.

The results from the hydrodynamical calculations which employ Glauber initial condition fluctuations and 4πη/s = 1 show relatively good agreement with the \(v_{2,3}(\Psi_{2,3})\) data. The exact statistical significance of these constraints should be determined through a global fit procedure, including a quantitative accounting of the breakdown of hydrodynamics in peripheral collisions, as well as of the systematics associated with the averaging of eccentricity fluctuations within the models. From our data it is already clear that the higher-order moment \(v_3\) should provide an important avenue for constraining different physical properties of the QGP.

In summary, we have presented participant event-plane \(\Psi_n\) correlations and differential measurements of \(v_n(\Psi_n)\) for \(n = 2, 3, 4\) for charged hadrons using the generalized event-plane method. The higher-order harmonic moments \(v_3(\Psi_3)\) and \(v_4(\Psi_4)\) and the nonzero correlations between higher-order event planes across a large rapidity gap of \(\Delta y/\geq 7\), indicate that the initial state has transverse geometry fluctuations. These fluctuations affect the generalized eccentricities, which are subsequently propagated in the hydrodynamic evolution of the plasma. The evidence includes (1) a lack of correlation between the measured event planes of order \(n = 2\) and 3 as predicted by Glauber modeling, assuming correlations of the event planes with the generalized eccentricity, (2) proper description of the shapes of the \(p_T\) dependence in the low \(p_T\) region by hydrodynamic calculations, and (3) agreement with several different initial state + hydrodynamic models across centralities for order \(v_n(\Psi_n)\). The combined results for \(v_{2,3}(\Psi_{2,3})\), together with initial hydrodynamic-model calculations now suggest that one of the important factors contributing to a large uncertainty in the extracted value of 4πη/s can be significantly reduced. For the limited set of models considered, 4πη/s ≈ 1 is favored.

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